Aloha ʻĀina Mathematics:

Honoring Resilience, Wisdom, and Cultural Identity in the Mathematics Classroom

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Aloha ʻĀina Mathematics reflects a connection between Aloha ʻĀina (love for the land) and the subject of mathematics, suggesting a pedagogical approach rooted in the values and context of Hawaiʻi. The term Aloha ʻĀina emphasizes a deep appreciation and care for the environment (Meyer, 2001). The framework presented in this study suggests an approach that integrates traditional ways of knowing into the teaching and learning of mathematics.

This study explores how teacher candidates applied and integrated a framework combining place-conscious (Gruenewald, 2003a, 2003b), culturally responsive (Gay, 2009; Ladson-Billings, 2014; Paris & Alim, 2014), and critical ethnomathematics education (Mukhopadhyay et al., 2009) grounded in Nā Hopena Aʻo (Lupenui et al., 2015). The study utilized a constructivist grounded theory design within sociocultural theory through a mixed methods analysis rooted in three guiding principles —pilina, kuleana, and pono. Surveys, student work, and observations within the study highlight the significance of building a solid student-teacher relationship and fostering place-consciousness to create a sense of belonging among students.

The current state of mathematics classrooms in Hawaiʻi, which largely adheres to Western-centric approaches, perpetuates a process of Americanization (Sai, 2021) that marginalizes the cultural heritage and experiences of the aboriginal people of Hawaiʻi. By maintaining traditional teaching methods that prioritize Eurocentric mathematical concepts, educators inadvertently contribute to the erasure of traditional ways of knowing and reinforce historical trauma. Embracing traditional ways of knowing in mathematics classrooms enriches students' educational experience and contributes to the broader decolonization of education in Hawaiʻi. By challenging the dominance of Western mathematics and acknowledging the diverse ways of understanding mathematical concepts, educators can dismantle systems and promote cultural revitalization and empowerment.

The transformation of mathematics classrooms in Hawaiʻi to incorporate traditional ways of knowing is a vital step toward honoring the resilience, wisdom, and sovereignty of Native Hawaiian communities. It represents a commitment to creating an educational system that celebrates cultural diversity, fosters inclusivity, and prepares all learners for a more equitable and inclusive future.

References

Gruenewald, D. A. (2003a). Foundations of place: A multidisciplinary framework for place-conscious education.

American Educational Research Journal, 40(3), 619-654.

Gruenewald, D. A. (2003b). The best of both worlds: A critical pedagogy of place. *Educational Researcher*, *32*(4), 3-12.
Gay, G. (2009). Preparing culturally responsive math teachers. In B. Greer, S. Mukhopadhyay, A.B. Powell, & S. Nelson-Barber (Eds.), *Culturally responsive mathematics education* (pp. 189-206). New York, NY. Routledge.
Ladson-Billings, G. (2014). Culturally relevant pedagogy 2.0: a.k.a. the remix. *Harvard Educational Review*, *84*(1), 71-84.
Lupenui, C. K., Sang, D.K., Seward, H., Lee, H., Walk, K., Benioini, K., Kawaʻaeʻa, K., Albert, L., Duarte, M.P., Zeng, M.P., Morris, M., and Kahumoku III, W. (2015). "Nā Hopena Aʻo HĀ Statements: BREATH." Retrieved from http://www.hawaiipublicschools.org/DOE%20Forms/NaHopenaAoE3.pdf
Meyer, M.A. (2001) "Our own liberation: Reflections on Hawaiian epistemology," *The Contemporary Pacific*, 13(1), pp. 124–148.
Mukhopadhyay, S., Powell, A. B., Frankenstein, M. (2009). An ethnomathematical perspective on culturally responsive mathematics education. In B. Greer, S. Mukhopadhyay, A.B. Powell, & S. Nelson-Barber (Eds.), *Culturally responsive mathematics education* (pp. 65-84). New York, NY. Routledge.
Paris, D., & Alim, H. S. (2014). What are we seeking to sustain through culturally sustaining pedagogy? A loving critique forward. *Harvard Educational Review, 84*(1), 85-100.
Sai, D. (2021). Setting the Record Straight on Hawaiian Indigeneity. *Hawaiian Journal of Law and Politics*, *3*, 6-72.

Uruuru Whenua: Using Cultural Symmetry to Illuminate Māori Wayfinding Practices in Pāngarau Classrooms.

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Spatial awareness and reasoning intertwine with Māori oral and visual narrative practices and daily rituals. To ensure the intergenerational transmission of mātauranga (Māori ways of knowing, being and believing), such as Māori wayfinding practices, Māori aspirations for schooling include living in a way that is recognisably Māori to Māori, while also accessing notions of academic success (Allen, 2023; Smith et al., 2021). However, due to the colonisation of Aotearoa New Zealand (NZ) by the British Crown, Māori have endured more than 100 years of education policy that invalidated mātauranga, including the exclusion of the Māori language from schooling (Trinick, 2015). Māori communities, academics, policymakers and teachers have agitated for change, causing considerable shifts in education policy in Aotearoa NZ since the 1970s, including how curriculum is developed.

As part of national curriculum and assessment programmes, the NZ Ministry of Education (MoE) now promotes epistemological parity for mātauranga (MoE, 2022). However, tensions between Māori aspirations for schooling and colonial schooling ideologies continue to shape curriculum development, including pāngarau (mathematics in Māori language instruction schools). This includes minimising mātauranga to promote the acquisition of curriculum achievement objectives imported from countries with colonial legacies, such as the UK.

National curriculum and assessment programmes have the power to shape the educational experiences of Indigenous students, potentially providing knowledge and skill-based economic advantages, but also perpetuating knowledge debt (Aikenhead, 2018). Over time, curriculum and assessment programmes have perpetuated Indigenous knowledge debt (Allen, 2023), limiting Māori students' opportunities to succeed *as Māori* (MoE, 2020). Using the Cultural Symmetry Framework (Meaney et al., 2022, Rau et al., 2022, Trinick & Allen, 2023), this doctoral study identified opportunities to improve students' access to mātauranga alongside the high-status curriculum area of pāngarau. Data collection methods included: a literature search, interviews with pāngarau experts, collaborative pāngarau lesson design and the anaysis of student work samples. The study identified the teaching of Māori wayfinding and spatial reasoning practices as an opportunity to support Māori aspirations for schooling within pāngarau curriculum development and classroom practices.

Keywords: Indigenisation, Indigenous Mathematics Education, Māori Education, Curriculum Development, Pāngarau.

References:

Aikenhead, G. (2018). Indigenous perspectives in school mathematics: From intellect to wisdom. In A. Kajander, J. Holm, & E. Chernoff (Eds.), Teaching and learning secondary school mathematics (pp. 39–49). Springer. https://doi.org/10.1007/978-3-319-92390-1_5

Allen, P. (2023). Uruuru Whenua: Using Cultural Symmetry to Rebalance Mātauranga and School Mathematics (Doctoral Thesis, The University of Auckland). ResearchSpace. https://hdl.handle.net/2292/65818

Meaney, T., Trinick, T., & Allen, P. (2022). Ethnomathematics in education: The need for cultural symmetry. In *Handbook of Cognitive Mathematics* (Vol. 1-2, pp. 191-219). doi:10.1007/978-3-031-03945-4_4

Ministry of Education. (2020). *Ka Hikitia – Ka Hāpaitia. The Māori Education Strategy (English)*. https://www.education.govt.nz/our-work/overall-strategies-and-policies/ka-hikitia-ka-hapaitia/ka-hikitia-ka-hapaitia-the-maori-education-strategy

Ministry of Education. (2022). *Change 2 – Equal status for mātauranga Māori in NCEA*. https://ncea.education.govt.nz/change-2-equal-status-matauranga-maori-ncea

Rau, C., Goulton, G., & Te Maro, P. (2022). *Hei Raukura mō te Mokopuna Strategy [English]*. Ministry of Education. https://kauwhatareo-live-storagestac-assetstorages3bucket-1w7r7v9dv6o7k.s3.ap-southeast-2.amazonaws.com/public/Uploads/Hei-Raukura-Mo-te-Mokopuna-English.pdf

Smith, L., Keepa, M., McKinley, E., Brewin, M., Corscadden, K., Doherty, W., Harris, M., Hill, D., Hohepa, M., Hoskins, T. K., Jenkins, K., Lee, J., Kidman, J., Penehira, M., Sadler, H., Stephens, C., Stewart, G., Tangaere, A. R., Taurere, M., . . . Williams, M. (2021). *Conversations about the curriculum. A collaborative think piece*. Auckland UniServices.

Trinick, A. (2015). *Te reo tātai: The development of a mathematics register for Māori-medium schooling* [Doctoral thesis, University of Waikato]. Research Commons. https://hdl.handle.net/10289/9593

Trinick, T., & Allen, P. (2023). Ngā Uruuru ao. Spatial Orientation Representations. In P. Te Maro, & R. Averill (Eds.), *Ki Te Hoe! Education for Aotearoa* (pp. 161-177). Wellington: NZCER Press. Retrieved from https://www.nzcer.org.nz/nzcerpress/ki-te-hoe-education-aotearoa

Extended Abstract **Author: Jerry Lipka**

Working Title: How Yup'ik Counting is a Pathway to Mathematics, Culture, and Creativity

Introduction

Counting numbers, numerals, and the base ten system evolved across different civilizations and languages over thousands of years. Undoubtedly, they were derived from the human body (Ifrah, 2000) and have become abstracted disembodied concepts. Although numbers are used daily, "they remain silent about their inner meanings" (Menninger,1969, p.117). However, Indigenous number words across the Americas (Closs, 1986) to Papua New Guinea (Pickles, 2009, Owens, 2000 & Wolfers, 1971) retain cultural and linguistic knowledge of their embodied counting words. However, assimilationist forces fueled by capitalism and Western schooling weaken these connections. This case study demonstrates how one group[1] of Yup'ik Elders, teachers, and academics creatively and collaboratively devised pedagogical practices through Yup'ik ways of communicating and expressing. We resurfaced some of the number words' underlying meanings by coordinating the physical enactment of counting in Yup'ik with their literal meaning. Over time, we viewed the Yup'ik base 20 and subbase 5 system as part of Yup'ik epistemology and cosmology.

A Case Example

This case describes the importance of deep community engagement for exploring, reviving, and renewing the mathematics contained in the Yup'ik language. Simultaneously, we explored ways of teaching counting and arithmetic compatible with school mathematics, such as focusing on place value. Today, this approach holds the promise of renewing and revitalizing the Yup'ik language and culture.

Key components of this case material derive from revisiting and reanalyzing video footage of meetings held in Alaska (Fairbanks and two Yup'ik villages) around 1990. These meetings represent approximately nine days of Yup'ik Elders, Yup'ik, non-Yup'ik teachers, academics (Indigenous and non-Indigenous), parents, and students working together.

One of our first meetings began with Joshua Phillip, a well-respected Elder and leader from Akiachak, who spoke about the importance of sovereignty. He urged the teachers to work together and support what we were about to engage in, developing curriculum materials to teach the Yup'ik counting system in a Yup'ik way. Shortly after that, he was seated on the floor with Henry Alakayak (a Yup'ik Elder and leader in this effort), Mary George (a bilingual teacher aide), and other group members. They demonstrated Yup'ik counting by using short sticks, creating a bundle of 5 sticks, and then bundling four groups of 5 sticks into a group of twenty. These were their initial culturally mediated tools and representations.

Beyond identifying regional dialectical differences in Yup'ik, a substantial amount of time was devoted to understanding how English and Russian loan words (Jacobson & Jacobson, 1995) affected counting and performing arithmetic operations in Yup'ik (Lipka, 1994). In a follow-up meeting with Elders, teachers, and consultants, the Yup'ik drum was used to teach counting patterns. The participants' concentration, joy, and laughter reflected conviviality and aspects of a Yup'ik cultural approach to teaching. During the drumming, the room refocused on an Elder who stated that "the drumming reminded her of a Yup'ik dance." This altered the trajectory of the meeting by creating more space to choreograph a Yup'ik way of counting by drumming, which included writing the song in Yup'ik and developing a numbers dance as a form of storytelling. Another group of Elders, teachers, and consultants worked on drumming and bundling a set of ones to make five, and four fives to make twenty, emphasizing grouping and exchange. This laid the groundwork for a culturally adaptive way of teaching counting and place value. Figure 1 is a representation of the Yup'ik base 20 system.

[1] Math in a Cultural Context (MCC) is a long-term project with Yup'ik Elders and teachers working jointly with academics from the University of Alaska Fairbanks and elsewhere.

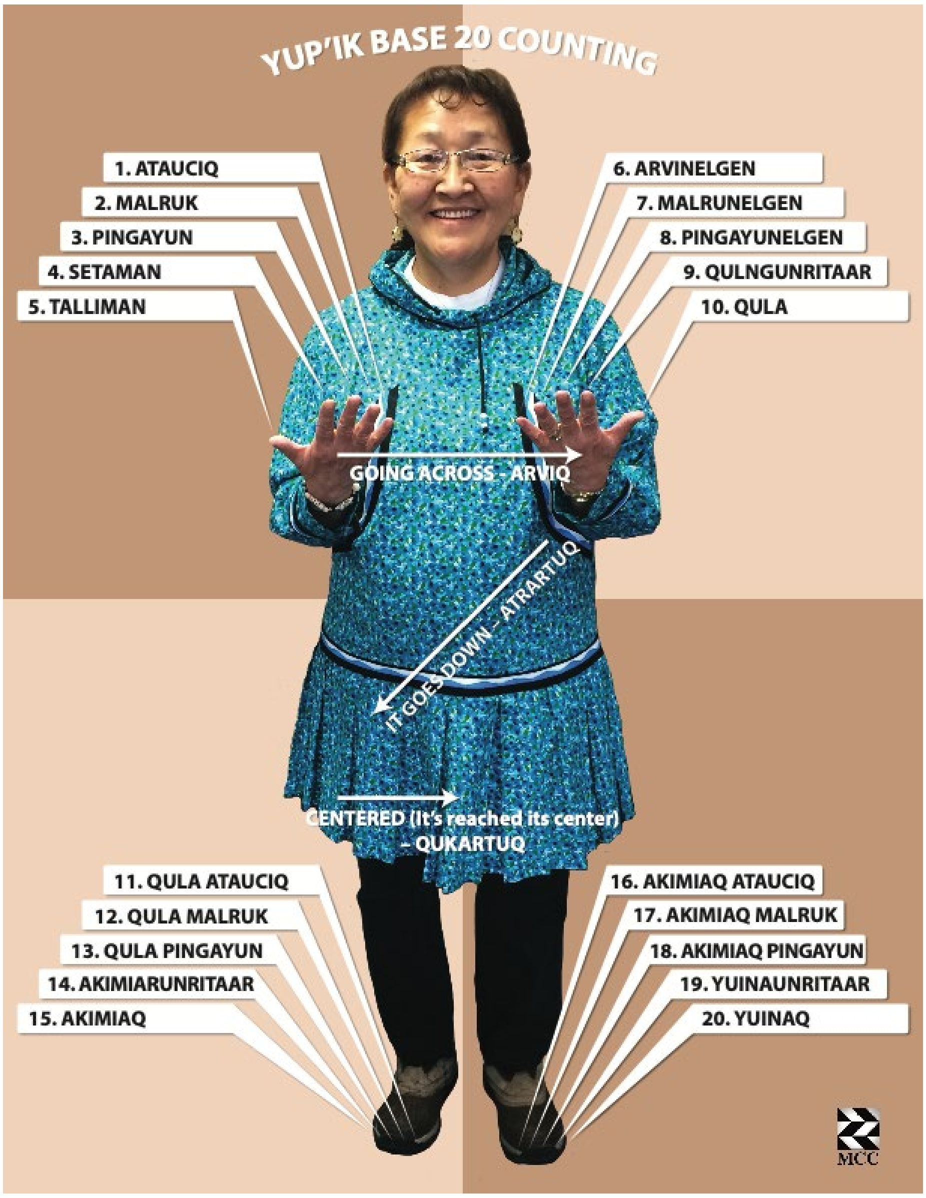

Figure 1: A representation of the Yup'ik counting system (credits: Jumiah Johnson created the poster which shows Evelyn Yanez; both worked for Math in a Cultural Context, University of Alaska Fairbanks)

Counting: Culture and Language

Years later, working with some of the same Elders and Yup'ik teachers, the Yup'ik base 20 sub-base five counting system emerged as much more than a Yup'ik equivalent to the Indo-European base ten system. There are structural differences in teaching arithmetic operations in Yup'ik versus English, most notably questions such as, how much is ten plus twenty? In Yup'ik, the question is the answer.

The literal meaning of some Yup'ik counting words includes spatial words such as *arvinelgen* (across), *qula* (above), and *atrartuq* (going down to the feet) (Eells, 1913); these words indicate movement, directionality, locations, and orientation. With these insights, the Yup'ik counting system was recast as a subset of their epistemological system. The counting words and their embedded actions and spatial imagery (above/below, sidedness, and crossing the center) underlie basic structures used by artisans.

Schooling

We developed *Going to Egg Island: Adventures in Grouping and Place Value* (Lipka, 2003), a second-grade school mathematics module implemented in many school districts in Alaska. The module's context, tools, pedagogy, and associated storybook reflected life in Yup'ik communities. We conducted a quasi-experimental study with Alaska Native and non-Native students in urban and rural settings, finding statistically significant results on students' math content knowledge (see Kisker et al., 2012).

Conclusion

This case example illustrates that long-term collaborative community-based work can expand ways of teaching counting, grouping, exchanging, and strengthening students' understanding of place value. Simultaneously, this long-term effort helped recover, document, and hopefully renew cultural knowledge.

References

Closs, M. P. (1986). *Native American mathematics*. University of Texas Press.

Eells, W. C. (1913). Number systems of the North American Indians. *The American Mathematical Monthly*, *20*(9), 263–272.

Ifrah, G. *(2000). The universal history of numbers.* London: Harvill.

Jacobson, S. A., & Jacobson, A. W. (1995). *A practical grammar of the Central Alaskan Yup'ik Eskimo language.* Alaska Native Language Center, University of Alaska Fairbanks.

Kisker, E. E., Lipka, J., Adams, B. L., Rickard, A., Andrew-Ihrke, D., Yanez, E. E., & Millard, A. (2012). The potential of a culturally based supplemental mathematics curriculum to improve the mathematics performance of Alaska Native and other students. *Journal for Research in Mathematics Education*, *43*(1), 75-113.

Lipka, J. (1994). Culturally negotiated schooling: Toward a Yup'ik mathematics. *Journal of American Indian Education*, pp. 14–30.

Menninger, K. (1969). *Number words and number symbols: A cultural history of numbers*. Courier Corporation.

Owens, K. (2000). Traditional counting systems and their relevance for elementary schools in Papua New Guinea. *Papua New Guinea journal of education*, *36*(1/2), 62–72.

Pickles, A. J. (2009). Part and Whole Numbers: an 'Enumerative' Reinterpretation of the Cambridge Anthropological Expedition to Torres Straits and its Subjects. *Oceania*, *79*(3), 293–315.

Wolfers, E. P. (1971). The original counting systems of Papua and New Guinea. *The Arithmetic Teacher*, *18*(2), 77–83.

Mathematics language-knowledge differentials and coloniality in the Philippines

Ivy Estrella, Paola Valero, Hendrik Van Steenbrugge

In a world ruled by the coloniality of power, codified power relations determine who includes and who is included (Mignolo, 2011). Coloniality pertains to the control of economy, authority, gender and sexuality, and subjectivities and knowledge (Ndlovu-Gatsheni, 2012; Mignolo, 2011). The universal epistemic 'Western' code assumes the idea that diverse knowledge is in modern or imperial languages, that the point of arrival of human history is Eurocentric modernity and thus, colonizes certain people by denying humanity, intelligence and knowledge. With this, there is a continuous fight in the form of what Mignolo (2011) calls 'options', over control of knowledge and authority from the state, business entities, and religious groups along with the fight over the freedom from that control. One of the options to fight control is decoloniality. The decolonial movement invites us to rethink and critique eurocentrism from the perspectives of the subaltern side and decolonize Eurocentric epistemology (Grosfoguel, 2013). It aims to delink from the western code and participate in the production of knowledge from the perspective of those who were left out (Mignolo, 2011; Walsh, 2013). The task of decoloniality is two folds: 1) to liberate the knowledge and becoming that the coloniality of power (knowledge and being) had prevented by questioning the foundation of the Western code or Eurocentric epistemology and ontology (Mignolo, 2018); and 2) to develop decolonial narratives that will legitimize decolonial ways of doing and living, by opening up ‘other’ options of logic of thinking, doing, and living (Veronelli, 2015).

Questioning the foundation of Eurocentric epistemology and ontology by revealing coloniality, is a crucial first step in the process of decoloniality. In the Philippines, coloniality is maintained through the organization of the educational system (Casambre, 1982). In mathematics education in the Philippines, coloniality operates through language(s) and forms of knowledge(s). In particular, mathematics is taught in English, a colonial language. The mother tongue languages of the students, Filipino and others, are only used as auxiliary languages of instruction. These languages get replaced by English as children progress in school. With the replacement of students’ mother tongue languages by English comes the replacement of mathematics which are part of work practices in communities by school mathematics. Hence, this study asks the question: *How do language(s) and mathematical knowledge(s) connect in educational practices to perpetuate and maintain coloniality in the Philippines?*

Based on an ethnographic study in classrooms and communities in a province in the Philippines, disparities between school mathematics practices and community mathematics practices were identified. In the school, lessons on telling time, and identification of solid figures were observed, while people in the community were interviewed about their jobs. From this, it was found that students are taught to read and estimate time in English but outside the classrooms, time is usually expressed in Spanish and Filipino. The interviews showed that people in the community or outside the school use body, items, and nature as basis of measurements as shown with measuring the size of fish through the number of fingers or length of the arms, measuring height of the water by using body parts and trees as basis, and describing the amount of bamboo in terms of money. The measurements are expressed using Spanish or Filipino language.

One episode that encapsulates the tension between school and community mathematics knowledge and practices took place in the lesson about solid figures. This is a 5th-grade lesson, taught in English and supported by mother tongue language, in which Euclidian geometry on solid figures are taught to the students. The students had to identify the solid figures outside their classroom, but within the school. In groups, they had to look for 10 objects and identify what solid figure they are. The groups were tasked to go to five stations —classroom, basketball court, canteen, flagpole and garden— and identify two objects in each. When the students were in the garden, one student kept on asking about a mango that he saw on the ground: 'What about the mango? What figure is this?' The teacher was with them in the garden but this student did not ask the teacher about it. Back in the classroom, the student did not ask the teacher. In the end, the students did not choose the mango because they could not identify what solid figure it is. Ivy (the observer) asked the teacher about the situation. According to her, a mango is an irregular solid figure and since it is not part of the elementary mathematics curriculum, it is ignored. From this vignette, it can be seen that students have experiences that are not part of the curriculum and these experiences are obliterated, and thus this knowledge is not recognized inside the classroom.

Looking at the society in terms of the disparity between occupations and jobs that are categorized as elementary occupation and high-end occupations, we can further see the role of language in mathematics education in the Philippines. Elementary occupations include laborers and workers which do not require highly valued qualifications or skills; while high-end occupations include professional workers which require highly valued qualifications and skills. From this, occupations and industries which belong to high-end require specific qualification and skills such as school mathematics skills, while elementary occupations require other, lower valued qualifications and skills, which, most often do not relate to school mathematics. Thus, it can be said that schools, and in particular, language used in mathematics, is a significant factor in the maintenance and perpetuation of coloniality in the Philippines, also associated to a hierarchy of people, knowledge, socio-economic status and occupations. The English language in tight connection with school mathematics knowledge continues to be the language- knowledge of domains of power and prestige as it creates a separation between the past and the present, the educated and the illiterate, the rich and the poor. It became the language-knowledge of the elite, while local languages and knowledges belong within the uneducated (Bustamante, 2016; Bernardo 2004).

Our classroom observations and interviews with people from the community have revealed that particular kinds of school mathematics languages and knowledges are privileged over the mathematics and languages of the communities. We understand this privileging as traces or signs of coloniality through language in elementary school mathematics classes. In so doing, we have worked toward questioning the foundation of the Western code, and opening up for other options of thinking, and doing mathematics, two central aspects of decoloniality.

References

Bernardo, A. B. (2004). McKinley's questionable bequest: Over 100 years of English in Philippine education. *World Englishes*, *23*(1), 17-31.

Bustamante, C. B. S. (2016). Renato Constantino's Discourses on Philippine Education as Post-Colonial: A Philosophical Reading. *The Research Journal of the College of Arts & Sciences*, 9.

Casambre, N. J. (1982). The Impact of American Education in the Philippines. *Educational Perspectives*, *21*(4), 7-14.

Grosfoguel, R. (2013). The Epistemic Decolonial Turn: Beyond political-economy paradigms1. In *Globalization and the decolonial option* (pp. 65-77). Routledge.

Mignolo, W. (2011). *The darker side of western modernity: Global futures, decolonial options*. Duke University Press.

Mignolo, W. (2018). The Conceptual Triad: Modernity/Coloniality/Decoloniality. In *On Decoloniality Concepts, Analysis, Praxis* (pp.135-152). Duke University Press.

Ndlovu-Gatsheni, S. J. (2012). Coloniality of power in development studies and the impact of global imperial designs on Africa. *Australasian Review of African Studies, The*, *33*(2), 48-73.

Veronelli, G. (2015). The coloniality of language: Race, expressivity, power, and the darker side of modernity. *Wagadu*, *13*(1), 108-134.

Where does the mathematics end and the culture and language start?

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"Where is the mathematics?" is one type of feedback experienced by many scholars investigating cultural and linguistic aspects in the mathematics classroom. Through a five-year participatory research project Matematihkkaoahpahus álgoálbmot- ja migrašuvdnaoktavuođas (MIM) we have approached cultural and linguistic aspects in mathematics education by analyzing positive experiences expressed by individuals and groups in their own stories and building on asset-based mathematics. When analyzing our data we have identified strong stories about students' experiences related to cultural and linguistic backgrounds. As our attention is directed towards mathematics teaching and learning, we are wondering if and how stories about culture and linguistic might be included in our hunt for storylines about mathematics. On the one hand, we are worried that there is no mathematics within these stories. On the other hand, we feel obliged to share these stories that our informants have bravely told us. Further, we wonder how these stories best could have been followed up during the interviews.

We will use examples from our data material of interviews with school leaders, becoming and practicing teachers, students and community members to illustrate our concern. We invite the participants of the conference to discuss the ethical and moral aspects of drawing the line between enough mathematics and too much culture and language and our responsibility as researchers.

The first example is from an interview with a student, Sverre, at a school in Northern parts of Norway. The theme for the interview is mathematics teaching, the preferences and thoughts of the students around teaching and learning mathematics. At the beginning of the interview the researchers/interviewer ask some questions about the background of the student and the following comes up:

> *Sverre: Jeg har kvensk på faren min sin side, men det er ikke så mye jeg kan bare telle til 3. Jeg gikk på kvensk før, men jeg droppet det. Bestefaren min kan kvensk, men pappa kan ikke så mye og jeg kan ikke så mye.*
>
> *I: Hvorfor sluttet du?*
>
> *Sverre: Det var på grunn av læreren. Vi var 3-4 stykker som sluttet og det var på grunn av at læreren. Det er lenge siden, men jeg husker vi fikk en ny lærer i kvensk og alle var veldig misfornøyde med henne. Hun var veldig sånn … jeg følte hun så ikke på oss som elever eller faktisk mennesker liksom, hun var veldig ekkel da, og jeg sluttet da, det var ganske mange som sluttet da.*
>
> *I: Så leit. Hun var kanskje ikke lærer så lenge da, eller?*
>
> *Sverre: Jeg tror ikke hun er lærer nå, så…*
>
> *I: Det var jo synd…men… Men vi er jo interessert i skole og matematikk. Trives du på skolen og matematikk?*

As we see, the interviewers keep to the plan and continue the dialogue around mathematics despite the shocking story shared from the student's experiences from the subject Kven language.

The second example is from an interview with a mother talking about her child. Again, the context is mathematics and culture, but in this interview the researchers open up for discussing only language and culture and the difficulties that the participants have experienced.

> *Hun forklarte at hun hadde brukt mye tid på å veilede hen bort fra alt det kvenske*, hun hadde ikke lyktes, men hun hadde hatt det som målsetning i 8.,9., og 10. Hun forklarte at det bare var en dialekt og ikke hørte til i skriftspråket. Vi syntes dette var veldig vanskelig, at hun gjorde det på denne måten. (Merete)*

*Kven language is taught together with Finnish language in Norwegian school. Merete is here explaining how the teacher want to guide the student towards Finnish language and away from Kven language.

The third example is from an interview with a becoming mathematics teacher, Emma:

> *Og så den her gruppa den er veldig glad i å skjule de svakhetene de har, sånn at du kan gå til en som sier at han skal ikke gjøre noe for han hater det, så leser du oppgaven og så …. Da kan han gjøre den. Jeg tror ofte at mange av dem også er underkommunisert at det ikke er oppdage dysleksi eller en eller anna vanske som ikke er oppdaga hos dem. Det blir bare … (Emma)*

In this example Emma is touching on whether or not the challenges of students from minoritized groups are seen or understood. Here Emma is explaining how reading the mathematics task out loud to the student enables him or her to solve the task. It is not mathematics that is the problem, it is the text.

Mathematics and mathematical knowledge as a part of a general knowledge base is linked to language and culture. We question how the future of the minoritized groups and their access to language and own culture affects their access to mathematics. Our data shows how children are denied the possibility to learn their Indigenous language in schools and that their background limits their access to mathematics in school, and most likely later in their career choices and professional work as an adult. Considering the example of Sverre: Can we investigate mathematics and culture and language in mathematics classrooms by only focusing on what happens in the subject mathematics and ignoring how the culture and language of the student is met in school in general?

Our research project started out with the following questions: Can mathematics contribute to more equality and social justice in school? And can the subject itself be enhanced by the diversity of languages and cultures? Our examples and excerpts although uttered in a mathematics educational context, are not only about mathematics education, but about language and culture. We wonder if when the informants are reflecting back, the cultural and linguistic challenges being closer connected to identities and “who we are” overshadows any mathematical challenges. And on the other hand, considering mathematics as a subject, can the more general experiences in context of own language(s) and culture(s) hinder, support or challenge learning in mathematics education.

Tangible Mathematics: String Figures and Body-Based Units of Measure as Ethnomathematical Systems

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Focus of the Paper

This paper synthesizes findings from two large cross-cultural studies exploring culturally embedded traditional and indigenous practices that have significantly shaped ethnomathematical thinking across societies worldwide. The primary focus of this paper is in understanding the origins, evolution, and implications of 1. *stringfigure games* (Kaaronen et al., 2024) and 2. *body-based measurement systems* (Kaaronen et al., 2023). These studies investigate how these practices are not only widespread across different cultures but also serve as cognitive tools that contribute to the development of mathematical and spatial reasoning. Owing to their exceptionally wide distribution-we have documented and analysed string figures across 92 societies, and body-based units of measure in 186 cultures-these practices serve as important case studies for understanding the emergence of mathematical thinking in humankind more broadly.

Theoretical Framework

The studies operate within a cultural and cognitive anthropological framework, emphasizing the role of practical ('tangible') mathematical activities in cognitive development and technological change. The research on string figures is grounded in theories of cultural transmission and innovation, whereas the study on body-based measurements draws on historical and ethnographic methods to trace the evolution of measurement systems. Both studies engage with niche construction theory to discuss how human interactions with cultural artifacts influence cognitive development. Theories of embodied cognitive science are also discussed, since both case studies illustrate how embodied practices (playing with string, measuring with one's body) can contribute to mathematical thinking, specifically through the manipulation of objects in the everyday environment.

Methodology

A variety of methods are utilized. The string figure study utilizes a novel knot theory-based methodology for the numerical coding of string figure designs, analysing 826 figures from 92 societies to explore patterns of innovation and cultural transmission. This has enabled us to computationally identify similar string figures between cultures. The body-based measurement research documents measurement units from 186 cultures, employing an ethnographic approach to understand the persistence and functionality of these measures in various technological domains. Both studies employ computational and cross-cultural analyses, offering insights into the global distribution of these practices and their cognitive-cultural implications.

Scientific and Cultural Context

These studies are situated within a broader discourse on how ethnomathematical practices can have notable impact on cultural change. The string figure research discusses the role of early human technologies in cognitive evolution, suggesting a deep ancestry for string games as a catalyst for mathematical thinking. It also provides grounds for rethinking humans' early relationships between algorithms, topology, and geometry. The body-based measurement research similarly discusses the deep ancestry of body-based measurement, illustrating also how such measurement systems have been used resourcefully, such as in the design of ergonomic equipment (from skis to kayaks). Since both studies are global reviews, documenting some widely shared (yet still locally variable) mathematical practices, insights from this work can have widespread application. Both string figures and body-based measurement systems play important roles in traditional and indigenous knowledge systems worldwide.

Importantly, as a part of these reviews, we have also stored and documented the indigenous concepts and terminology related to these practices m two large datasets (https://doi.org/10.17605/OSF.IO/SKZVP and https://doi.org/10.17605/OSF.IO/FEGVR; open CC- BY license, available for public reuse). These may be useful resources for further exploring the relations between language, cultural practices, and indigenous mathematical concepts. To further connect this research to indigenous language issues in mathematics education, I propose several practical applications of the documented databases. Educators can use such concepts and terminologies to develop culturally relevant mathematics curricula, helping students understand the connection between linguistic heritage and mathematical principles. For example, the string figure database illustrates how local ecologies (landscapes, animals, etc.) are referred to in the mathematical practice of string figure making, and also how mathematics relates to oral traditions and storytelling. These linguistic resources can help us understand how mathematics is embodied and enacted in everyday cultural traditions by highlighting how different cultures conceptualize mathematics, often using bodily metaphors, enacted through tangible and embodied practices. Additionally, both researchers and teachers can use these resources to study or illustrate how different cultures encode mathematical knowledge in language, uncovering both the similarities and differences of cultural- cognitive approaches to mathematical problems.

References

Kaaronen, R. 0., Manninen, M.A., & Eronen, J. T. (2023). Body-based units of measure in cultural evolution. *Science, 380(6648),* 948-954. https://doi.org/10.1126/science.adfl 936

Kaaronen, R. 0., Walsh, M. J., Henrich, A. K., Wisher, I., Miu, E., Manninen, M.A., Eronen, J. T., & Riede, F. (2024). *A global cross-cultural analysis of string figures reveals evidence of deep transmission and innovation.* https://doi.org/10.31219/osf.io/yxpg8

Geresjohtima vektorat –

Ánná Káisá Partapuoli

Dukkus lea čađahuvvon giđa 2021 mu masterbargun lektoroahpus Romssa universitehtas. Barggu váldo dutkangažaldat lea «Mo geresjohtima guorahallan matematihkkaoahpahusas váikkuhivččii vektordoahpaga ipmirdeapmái?». Vástidit dán lei dárbu vuos oahppat eanet geresjohtima birra, oainnusmahttit geresjohtima matematihkalaš jurddašeami ja gávnnahit mo dát vástidivččii vektoriidda. Dán dihte lei dárbu álggus guorahallat vuollegažaldagaid, mat leat «Mo leat geresjohtima kultuvrralaš, gielalaš ja matematihkalaš bealit, ja mo dát leat čadnojuvvon oktii?» ja «Mo geresjohtima matematihkka sáhttá oainnusmahttojuvvot ja dulkojuvvot vektormatematihkkan?». Dáid golbma gažaldaga guorahalan iežan konferánsačállosis.

Dáhtačoaggin lea čađahuvvon komperatiiva case-dutkamin mas lean jearahallan guokte sámi geresduojára gerresa duddjoma ja geavaheami birra. D'Ambrosio (1985,1997,1999) etnomatematihka teoriijaiguin vuođđun lean geavahan kultuvrralaš symmetriija (Trinick, Meaney & Fairhall, 2016) analyseret jearahallamiid, ja dasa lassin etnomatematihkalaš QRS-systema (Barton, 1999) oainnusmahttit matematihka. Nu lean gávdnan geresjohtima kultuvrralaš, gielalaš ja matematihkalaš beliid, ja maid identifiseren vektormatematihka geresjohtimis. Dán analysa lean geavahan evttohit oahpahusa mas geresjohtin lea vuođđun, ja mas ulbmil lea ipmirdit vektordoahpaga. Nu lean ožžon vejolašvuođa vástidit dutkangažaldagaid.

Dutkamis lean gávnnahan ahte geresjohtimii gullá dárkilis, árbevirolaš máhttovuogádat, sihke kultuvrralaččat, gielalaččat ja matematihkalaččat. Dán matematihkalaš máhttovuogádaga livččii vejolaš fuomášit ja guorahallat guorahalli matematihkkaoahpahusas. Dát matematihkkaoahpahus livččii vuođđuduvvon sámi kultuvrii ja gillii. Livččii vejolaš čatnat oahpahusas ilbman matematihkalaš jurddašeami vektordoahpaga ipmirdeapmái, ja nu ládje nannet vektordoahpaga vuođđoipmárdusa.

Gáldut / referanser

Barton, B. (1999). Ethnomathematics and philosophy. Zentralblatt fur Didaktiker Mathematik, 31(2), 54–58.

D’Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. For the Learning of Mathematics, 5(1), 44-48.

D’Ambrosio, U. (1997). Where does ethnomathematics stand nowadays?. For the Learning of Mathematics, 17(2), 13-17.

D’Ambrosio, U. (1999). Introduction. Ethnomathematics and its First International Congress. ZDM The International Journal on Mathematics Education, 31(2), 50-53.

Trinick, T., Meaney, T., & Fairhall, U. (2016). The relationship between language, culture and ethnomathematics. Journal of Mathematics and Culture, 10(2), 175–191.

How and why do you go herringbone with skis?

Anne Birgitte Fyhn, Sara Katrine Aleksandersen, Ole Einar Hætta

The Sámi University of Applied Sciences (SUAS) is a Sámi higher education institution where mathematising cultural practices is part of the education. Mathematising is practiced as described by Freudenthal (1991), as an activity that contrasts concretising. Birkely (1994) refers written sources about Sámi skiers back in 552 AD, so skiing is part of Sámi culture. The teacher educator Anne and the pre-service teacher Sara present the start of developing a mathematics teaching unit where the cultural practice is skiing. When learning to ski, you start with your skis in a parallel position and move them forward one at a time. When you go uphill, you angle your feet with the toes outwards to prevent sliding backwards, as Figures 1 and 2 show. This is called *fiskebein* in Norwegian because your skis create a herringbone pattern in the snow. Anne invited Sara to a mathematics skiing lesson, to investigate how 'herringbone' could work as introduction to angles. The focus was how and why to go herringbone and how to explain it in Sámi. Sara uses the Norwegian term *fiskebein*, and similarly she knows how to plough downhill, without knowing any Sámi word for it.

Figure 1. Čiehka (corner)

Figure 2. Herringbone

Figure 3. Ploughing

When a hill's steepness increases, you must increase the angle between your skis. This is common sense among skiers also before school age, but it is usually communicated in body language without words. We intend to shed light on the research question: How can the skiing technique 'to go herringbone' contribute to Sámi mathematics education? Sara and Anne will present a practical approach to the teaching of angles through mathematising the use of skis.

The teaching of angles

Gjone and Nortvedt (2001) carried out a multiple-choice test of Norwegian six-graders' geometry knowledge. Their study indicates that a widespread idea among students is that a large angle has long arms. This finding is in line with Munier and Devichi's (2024) point, that a main obstacle in the construction of the angle concept is that children believe the angle's size is determined by the length of its arms. Gjone and Nortvedt's (2001) finding strongly indicates that there is a need for a focus on Norwegian students' understanding of the concept of 'angle' in primary school's early years. The Norwegian teaching tradition is to introduce angles by measuring, as for instance Ministry of Education and Research (1996) shows. Freudenthal (1991) warns against the measure approach to angles, because "[i]n order to measure angles, one has to subdivide angles. Subdividing angles can be confused with subdividing lengths or areas" (p. 363). Mitchelmore and White (2000) found reasons to believe that angles where both arms are visible are the easiest angles to grasp for children. This paper intends to contribute to how the concept of angle can be introduced if the teaching should be based on Sámi culture, language, and social life. We highlight oral descriptions of activities, which is in line with Freudenthal's (1991) point, that name giving is the first step towards consciousness, regarding space and the bodies around us.

A skiing approach to angles

Sara immediately showed how she used her skis uphill and downhill. She had no idea beforehand of what 'angle' could have to do with skiing. She said the Sámi word *čiehka* (corner) made more sense than the Norwegian *vinkel* and in Figure 1 she shows how to shape a corner by her skis. Here both arms of the angle are visible, as suggested by Mitchelmore and White (2000). Sara immediately could show how she is 'going herringbone', see Figure 2; that is basic knowledge to her. However, it was harder for her to find words for telling how she does it. She used the Norwegian term *fiskebein*; despite the fact that skiing is an integrated part of Sámi culture, she did not know any Sámi word for this. Freudenthal (1991) points out that whether something is common sense is witnessed by how it is verbalised in common language. Anne was introduced for the North Sámi verb *spiehččut* for herringbone in Unjárga/Nesseby (Hansen & Fyhn, 2020). She asked Sara about this word. However, Sara is from a different North Sámi dialect area, and *spiehččut* was a new word to her. We found very limited written sources about Sámi skiing terms. Sara got one hit when searching the Sámi Facebook group *Ártegis Ságat* (strange talk) for *fiskebein*: A question from November 2023 about Sámi words for *fiskebein* got 66 comments and several suggestions. This shows that there is not a common agreement about one word. *Hárrut,* which according to Nielsen (1979) means to ski with arms and legs wide apart, was one of the suggestions. Nielsen's large Sámi dictionary presents no Sámi words for uphill skiing, despite that skiing was and is common in the Sámi areas where his work was conducted.

Sara's comments to this practical and physical approach to mathematics was positive. She found the learning arena to be pleasant. She said that meeting outdoor teaching can be chaotic, but it feels safer to try out with students something you have experienced yourself. The investigation of whether herringbone skiing could work as an introduction to the teaching of angles, caused Sámi language investigations and search for hidden words. This search has to be continued. This is in line with the cultural symmetry framework that is developed for Māori mathematics education, (Meaney et al., 2022). One point of this framework is that when cultural practice, culture, and language is basis for the teaching before mathematics is focused, then language revitalisation is one expected outcome.

References

Birkely, H. (1994). *I Norge har lapperne først indført skierne*. Iđut.

Freudenthal, H. (1991). *Revisiting mathematics education. China Lectures.* Kluwer Academic Publishers.

Gjone, G., & Nortvedt, G. (2001). *Kartlegging av matematikkforståelse. Veiledning til geometri F og I.* Directorate for Primary and Secondary Education.

Hansen, L. & Fyhn, A. B. (2020). Sáhtá go sabehiiguin hervet? B. A. Bals Baal, L. Baal & K. Pedersen (Eds.), *Sámi kulturipmárdus mánáidgárddiin ja skuvllain* (s. 22–27). htps://kunstkultursenteret.no/wp-content/uploads/2021/01/Samisk-kulturforstaelse-NordsamiskNorsk.pdf

Meaney, T., Trinick, T. & Allen, P. (2022). Ethnomathematics in Education: The Need for Cultural Symmetry. In M. Danesi (Ed.), *Handbook of Cognitive Mathematics* (pp. 191–218). Springer. https://doi.org//10.1007/978-3-031-03945-4_4

Ministry of Education and Research (1996). *Læreplanverket for den 10-årige grunnskolen*. Nasjonalt Læremiddelsenter.

Mitchelmore, M. C. and White, P. (2000). Development of angle concepts by progressive abstraction and generalisation. *Educational Studies in Mathematics, 41*, 209–238. https://doi.org/10.1023/A:1003927811079

Munier, V. & Devichi, C. (2024). Teaching and learning angles in elementary school: physical versus paper-and-pencil sequences. *International Journal of Mathematical Education in Science and Technology, 5*(6), 1420–1443. https://doi.org/10.1080/0020739X.2022.2108515

Nielsen, K. (1979). *Lapp Dictionary. Based on the dialects of Polmak, Karasjok and Kautokeino.* Volume III N–Æ. Universitetsforlaget. Original work published 1932–1962.

Hearggástallan ja árbevirolaš mihtidanvuogit

Čállit: Kristine Nystad ja Maret Gaup Eira

Dán artihkkalis leat sámi árbevirolaš mihttuid ovdamearkkat.. Árbevirolaš mihtideamis adnojit olbmo lahttut mihttun. Iešguđetge kultuvrrain miehtá máilmmi leat vuogit mo gorutlahtuiguin mihtidit. Movt sámit geavahit gieđaid ja julggiid mihttun, gullá sámi árbevirolaš máhttui ja birgemii. Dovdat ja diehtit min kulturárbbi lea dehálaš máhttu mas gánnáha vara váldit ja oahpahit boahttevaš buolvvaide. Mihtideapmi lea gealbomihttomearri matematihkkaoahpus vuođđoskuvlla álgodásis. Vuođđoskuvlla álgodási oahppit galget geavahit sihke standardiserekeahtes ja standardiserejuvvon mihtuid veahkkin.
Heargi lea leamaš dehálaš fievru sápmelaččain ovdal go mohtorfievrrut váldojuvvojedje adnui. Herggiid atne vuojánin, muoraid geaseheapmái ja ráidduin sihke jođidettiin ja gávpemátkkiin. Hearggit leat maid geavahuvvon noađđeheargin/noađđeguoddin ja gilvovuojánin. Dán dutkama ulbmil lea iskat mat árbevirolaš standardiserekeahtes mihtut hearggástallamis adnojit ja movt dat adnojit. Olbmo lahttomihtot leat heivehuvvon ráhkadit mihtuid mat adnojit hearggástallanoktavuođas nugo leaŋggaide, bákkiide, spagáide ja nu viidáset. Dát mihtidanvuohki lea buorre go dat lea álo mielde go lea vánddardeamen ja de sáhttá ávdnasiid maid ohcat.. Nubbi ulbmil lea guorahallat olbmo ja bohcco oktavuohta ja árvvuid mat vuhtiiváldit herggiid čálggu. Strukturrakeahtas ságastallama bokte diehtoaddiiguin geat dovdet hearggástallama, lea áigumuš oažžut ovdan árvvuid, estehtalaš jurddašeami ja doahpagiid. Hearggis lea ávki ja árvu ja dainna meannuda govttolaččat, juoga mii mihtidanvugiin boahtá ovdan. Čálus ii váldde vuhtii miehtá Sámi perspektiiva go guoská gorutmihtuid geavaheapmái. Diehtoaddit leat Guovdageainnus válljejuvvon. Guovdageainnus lea árbevirolašvuohta ain ealás sihke duojis, boazodoalus ja eará árbevirolaš bargguin.

Referánssat

Boine, M. (2024). Ságastallan Mari Boiniin NRK prográmmas *Sámi álbmotbeaivi* guovvamánu 6. beaivvi. NRK. Viežžan cuoŋománu 10. beaivvi dás https://tv.nrk.no/serie/samenes-nasjonaldag/2024/SAPP63007024/avspiller
Buljo, K. M. E. & International Centre for Reindeer Husbandry. (2012). *Gámagoarrun*. Guovdageaidnu: Riikkaidgaskasaš boazodoalloguovddáš.
Fyhn, A. B. & Nystad, K. (2014). Sami Culture and Values – A Study of the National Mathematics Exam for the Compulsory School in Norway. *Interchange 44, 349–367.* https://doi.org/10.1007/s10780-013-9201-3
Fyhn, A. B., Eira, E. J. S., Hætta, O. E., Juuso, I. A. M., Skum, E. M., Hætta, S. I., Sabbasen, B. K. S., Eira, E. H. and Siri, M. T. (2016). Fra kultur til matematikk – eksempelet lávvu, *Tangenten*, 27(3), 2–7. https://tangenten.no/wp-content/uploads/2021/12/fyhn.pdf
Gaup, A.Turi & Gaup, M. A. (1996). *Heargedápman*. Guovdageaidnu: Sámi oahpahusráđđi. Viežžan cuoŋománu 24.beaivvi 2024 dás nb.no.
Hætta, I. L. (1998). *Mun, moai, mii: fáddágirji. Min rumaš ja dearvvašvuohta. Min biebmu ja min biktasat*. Porsanger: Iđut.
Jannok Nutti, Y. (2007). *Matematisk tankesätt inom den samiska kulturen. Utifrån samiske slöjdares och renskötares berättelser* [Lisensiáhttadutkkus]. Institutionen för pedagogik och lärande, Luleå tekniska univeristet.
Kaaronen, R. O., Manninen, M. A. & Eronen, J. T. (2023). Body-based units of measure in cultural evolution. – *Science, 380*, 948–954(2023). https://doi.org/10.1126/science.adf1936
Lipka, J. Shockey. & Adams B. (2003). Bridging Yup'ik ways of measuring to Western mathematics. – Clements, D. H., & Bright, G. W. (doaimm,), *Learning and teaching measurement: 2003 yearbook*.

Reston, Va: National Council of Teachers of Mathematics.
Nilsen, S., Joks, S. & Andersen, S.. (2022). Lubmen- ja mannenpráksisat Porsáŋggus – mo mearrasámi guovllu olbmot dádjadit iežaset birrasiin. –*Sámi dieđalaš áigečála*, 2022, 7–30. https://doi.org/10.7557/sda.7033
Nystad, K. (2003). Geometri og tradisjonelle målenhter. – *Et faglitterært orkester. Antologi*. Rapport 5/2003. Høgskolen i Vestfold.
Oahpahusdirektorahtta - Utdanningsdirektoratet (2019a). *Læreplan i matematikk 1.-10.trinn*. https://data.udir.no/kl06/v201906/laereplaner-lk20/MAT01-05.pdf?lang=sme
Partapuoli, A. K. E.. (2021). *geresskálla mađiid miel Mo geresjohtima guorahallan váikkuhivččii vektordoahpaga ipmirdeapmái* Luonddudiehtaga ja teknologiija fakultehta Matematihka ja statistihka instituhtta Matematihka lektoroahpu masterbargu – MAT-3907 – miessemánnu 2021
Somby, D. & Utsi, Q. (2023). *Etnomatematihkka: Muhtin mihtidanvuogit fanasdahkamis.* Billávuotna: Mearrasiiddas Billávuonas. YouTube – viežžan cuoŋománu 20. beaivvi 2024 dás https://www.youtube.com/watch?v=vPTMVJd3TiQ